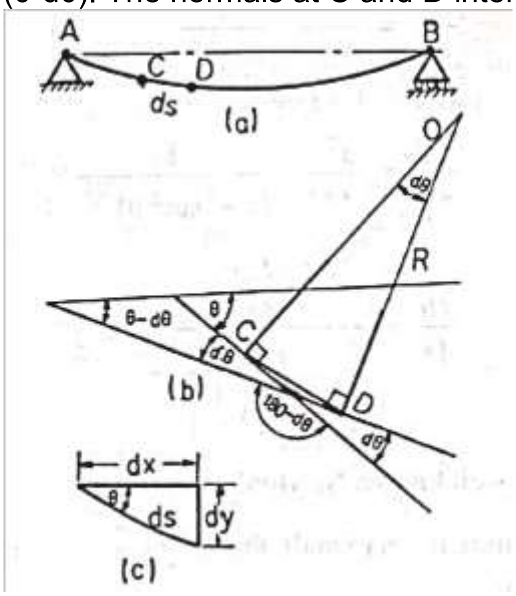
Chapter: 4
Deflection:

In certain situations it becomes necessary to design a machine component or a structure to minimize deflection. Thus it becomes essential to determine the deflection of the member. Also for statically indeterminate or fixed beams it becomes necessary to determine the slope and deflection at salient points.

Differential Equation of the Deflection Curve or (Relation between Bending moment, slope and Deflection.)

Consider a beam AB which takes the curved shape as shown in Fig(a). Consider an elementary length CD equal to ds of the beam. Let the tangent to the elastic curve at C makes with the x-axis of the beam an angle θ. The angle at D will decrease and let it be (θ-dθ). The normals at C and D intersect at O.



If R is the radius of curvature of the bent beam, then OC = OD = R

And ∠COD = dθ

∴CD = ds = R.dθ

$\frac{1}{R} = \frac{d\theta}{ds}$

Now from figure, tanθ $= \frac{dy}{dx}$; where y = deflection of the beam

Differentiating with respect to s, we get, $\frac{d}{ds} tan\theta = \frac{d}{ds}(\frac{dy}{dx})$

$Sec^2\theta . \frac{d\theta}{ds} = \frac{d^2y}{dx^2} . \frac{dx}{ds}$;

Now $\frac{dx}{ds}$ = cosθ

$Sec^3\theta.\frac{d\theta}{ds} = \frac{d^2y}{dx^2}$.

$$\frac{d\theta}{ds} = \frac{d^2y}{dx^2}.\frac{1}{sec^3\theta} = \frac{d^2y}{dx^2}.\frac{1}{\left(1+tan^2\theta\right)^{\frac{3}{2}}}$$

$$\frac{d\theta}{ds} = .\frac{\frac{d^2y}{dx^2}}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}} = \frac{1}{R}$$ ……….. Newton's formula for curvature

If the curvature is small, then $\frac{dy}{dx}$ is also small and its square is negligible, hence

$$\frac{1}{R} = \frac{d^2y}{dx^2}.$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{1}{R} = -\frac{d\theta}{ds} = -\frac{M}{EI} = -\frac{d^2y}{dx^2}$$

$$M = -EI\frac{d^2y}{dx^2}$$

The bending moment has been taken as positive if tension is caused in the bottom fibre of the beam. The positive bending moment should cause the positive curvature. The curvature is taken as positive if the centre of curvature O, is above the bent curve of the beam as shown in Fig (b). If the curvature is positive, the angle θ decreases in going from C to D, hence with proper sign.

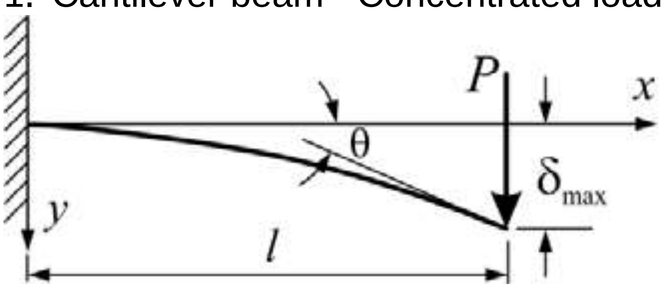
Slope of the beam, $\theta = \frac{dy}{dx} = -\int \frac{M}{EI} dx$

Deflection, $y = -\int \int \frac{M}{EI} dx \, . dx$

Shear force, $F = \frac{dM}{dX} = -EI.\frac{d^3y}{dx^3}$

And uniform load, $w = -EI.\frac{d^4y}{dx^4}$

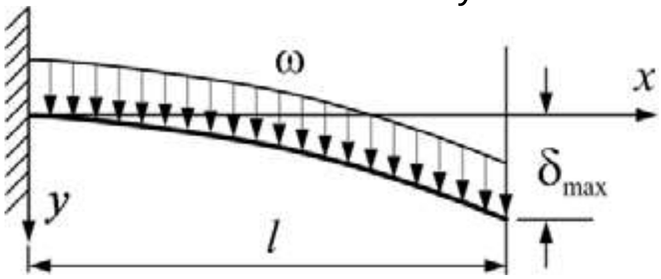
1. Cantilever beam - Concentrated load P at free end:



Slope at free end, $\theta = \frac{Pl^2}{2EI}$; Maximum deflection $\delta_{max} = \frac{Pl^3}{3EI}$

Deflection at any section in terms of x, $\delta_x = \frac{Pl^2}{6EI}(3l - x)$

2. Cantilever beam – Uniformly distributed load w(N/m)

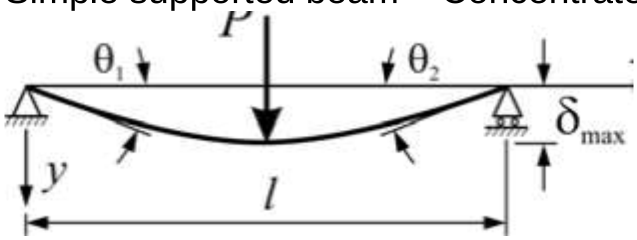


Slope at free end, $\theta = \frac{wl^3}{6EI}$

Maximum deflection $\delta_{max} = \frac{wl^4}{8EI}$

Deflection at any section in terms of x, $\delta_x = \frac{wx^2}{24EI}(x^2 + 6l^2 - 4lx)$

3. Simple supported beam – Concentrated load P at the centre

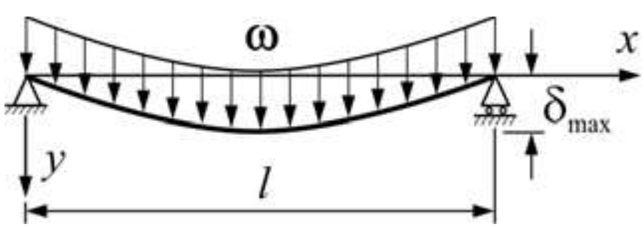


Slope at ends, $\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$

Maximum and center deflection $\delta_{max} = \frac{Pl^3}{8EI}$

Deflection at any section in terms of x, $\delta_x = \frac{P.x}{12EI}(\frac{3l^2}{4} - 6l^2)$ for <0<x<$\frac{l}{2}$

4. Simple supported beam – Uniformly distributed load w(N/m)



Slope at ends, $\theta_1 = \theta_2 = \frac{wl^3}{24EI}$

Maximum and center deflection $\delta_{max} = \frac{5wl^4}{384EI}$

Deflection at any section in terms of x, $\delta_x = \frac{w.x}{24EI}(l^3 - 2lx^2 + x^3)$