Design of a Shaft

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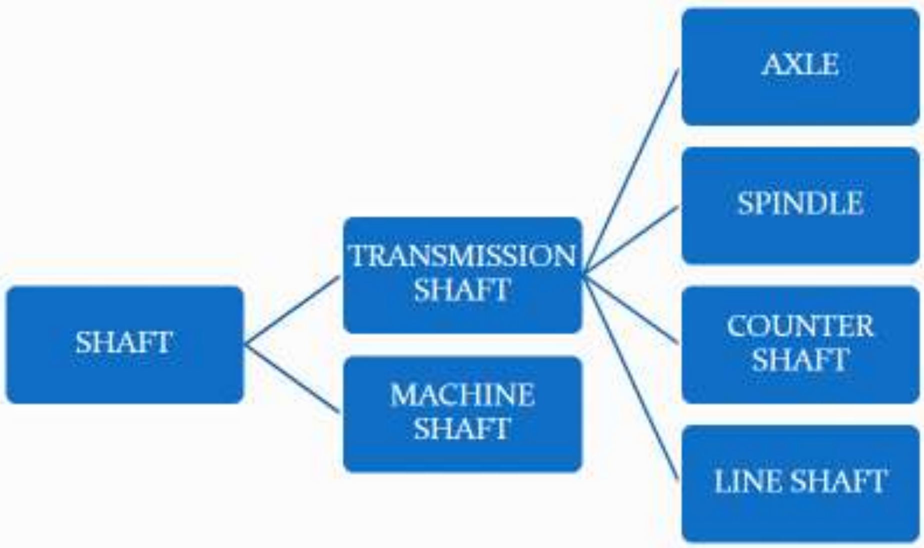
DESIGN OF SHAFT

WHAT IS A SHAFT?

Shaft is a common machine element which is used to transmit rotary motion or torque. It generally has circular cross-section and can be solid or hollow

Sr.No.	Shaft	Axle	Spindle
1	It is rotating element element of a machine	It is non-rotating or stationary element of machine	It is a circular component which revolves on something or something revolves on it
2	Shaft transmit torque as well as rotational motion	Axle is used to support the elements like wheels, pulleys, brake drums etc	Spindle is driving shaft, like spindle of drilling machine which carries the tool.
3	Shaft supports the driven member	Axle is only used as a load carrying member.	Spindle doesnot supports any other member
4	Examples :- Machine shaft, propeller shaft of automobile	Examples:- Front axle of automobile, Railway wagon axle which supports both wheels.	Examples: Lathe machine spindle, drilling machine spindle

Classification of shafts



Requirements of material for a shaft

1. Shaft material should have high strength.
2. Shaft material should have good machinability.
3. Shaft material should have low notch sensitivity factor.
4. Shaft material should have good heat treatment properties.
5. Shaft material should have high wear resistant properties.

MATERIAL:

General transmission shafts – medium carbon steels such as 30C8 and 40C8.

For high strength - high carbon steels such as 45C8 or 50C8.

for making transmission shafts 16Mn5Cr4, 16Ni3Cr2, 40Ni6Cr4Mo2.

Alloy steels have higher in strength, hardness and toughness.

Commercial shafts -low carbon steels

Design of Shafts

- In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to torque

(b) Shafts subjected to bending moment

(c) Shafts subjected to combination of torque and bending moment

(d) Shafts subjected to axial loads in addition to combination of torque and bending moment

Shafts Subjected to Torque

Maximum shear stress developed in a shaft subjected to torque is given by, $\tau = \frac{T\,r}{J} \leq [\tau]$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation

$= \frac{\pi d^4}{32}$ or solid shafts with diameter d

$= \frac{\pi (d_o{}^4 - d_i{}^4)}{32}$ for hollow shafts with d_o and d_i as outer and inner diameter.

r = Distance from neutral axis to the outer most fibre = d/2 (or $d_o/2$)

Shafts Subjected to Bending Moment

Maximum bending stress developed in a shaft is given by,

$$\sigma_b = \frac{M\,y}{I} \le [\sigma_t]$$

where M = Bending Moment acting upon the shaft,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation

$\frac{\pi d^4}{64}$= for solid shafts with diameter d

$\frac{\pi (d_o{}^4 - d_i{}^4)}{64}$ = for hollow shafts with d_o and d_i as outer and inner diameter.

y= Distance from neutral axis to the outer most fibre = d / 2 (or $d_o/2$)

Shafts Subjected to Combination of Torque and Bending Moment

$$\tau = \frac{T\,r}{J} = \frac{T\,\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{16\,T}{\pi d^3}$$

$$\sigma_b = \frac{M\,y}{I} = \frac{M\,\frac{d}{2}}{\frac{\pi}{64}d^4} = \frac{32\,M}{\pi d^3}$$

Maximum Shear Stress Theory

Maximum shear stress is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \sqrt{\left(\frac{16\,M}{\pi d^3}\right)^2 + \left(\frac{16\,T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \le [\tau]$$

$\sqrt{M^2 + T^2}$ is called equivalent torque, T_e, such that

$$\tau_{max.} = \frac{T_e\,r}{J} \le [\tau]$$

- **Maximum Principal Stress Theory**

Maximum principal stress is given by,

$$\sigma = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \frac{16\,M}{\pi d^3} + \sqrt{\left(\frac{16\,M}{\pi d^3}\right)^2 + \left(\frac{16\,T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right] \leq [\sigma_t]$$

$\left[M + \sqrt{M^2 + T^2}\right]$ is called equivalent bending moment, M_e, such that

$$\sigma = \frac{M_e\,y}{I} \leq [\sigma_t]$$

A.S.M.E. Code for Shaft Design

According to A.S.M.E. code, the bending and twisting moment are to be multiplied by factors k_b and k_t respectively, to account for shock and fatigue in operating condition.

$T_e = \sqrt{k_b M^2 + k_t T^2}$ and $M_e = \left[k_b M + \sqrt{k_b M^2 + k_t T^2}\right]$

	k_b	k_t
Gradually applied load	1.5	1.0
Suddenly applied load (minor shock)	1.5-2.0	1.0-1.5
Suddenly applied load	2.0-3.0	1.5-3.0

Introduction

Key is a machine element which is used to connect the transmission shaft to rotating machine elements like pulley, gear, sprocket or flywheel. Keys provide a positive means of transmitting torque between shaft and hub of the mating element. A slot is machined in the shaft or in the hub or both to accommodate the key is called keyway. Keyway reduces the strength of the shaft as it results in stress concentration.

Keys are made of ductile materials. Commonly used materials for a key are hardened and tempered steel of grades C30, C35, C40, C50 and 55Mn75 etc. Brass and stainless keys are used in corrosive environment. Factor of safety of 3 to 4 is generally taken on yield strength.

Types of Keys

Common types of keys are:

1. Sunk keys 2. Saddle keys 3. Tangent keys 4. Round keys 5. Splines

Sunk Keys

A sunk key is a key in which half of the thickness of key fits into the keyway in the shaft and half in the keyway of the hub. The sunk keys are of the following types:

Rectangular sunk key: It is the simplest type of key and has a rectangular cross-section. A taper of about 1 in 100 is provided on its top side. Rectangular sunk key is shown in Figure 1.

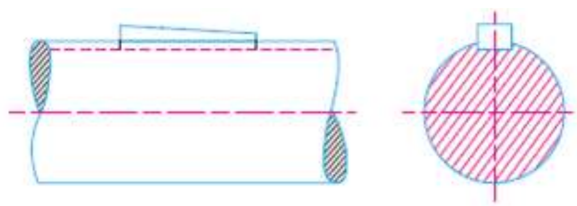


Figure 1 Rectangular Sunk Key

Square sunk key: Rectangular sunk key having equal width and thickness is called square sunk key.

Parallel sunk key: If no taper is provided on the rectangular or square sunk key, it is called parallel sunk key i.e. it is uniform in width and thickness throughout. It is used where the pulley, gear or other mating piece is required to slide along the shaft.

Gib-head key: It is a rectangular sunk key with a head at one end known as gib head, which is provided to facilitate the removal of key. Gib Head key is shown in Figure 2.

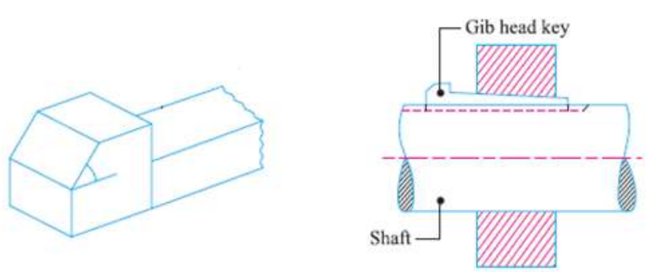


Figure 2 Gib Head Key

Feather key: Feather key is a parallel key made as an integral part of the shaft with the help of machining or using set-screws. It permits axial movement and has a sliding fit in the key way of the moving piece. Feather keys are shown in Figure 3.

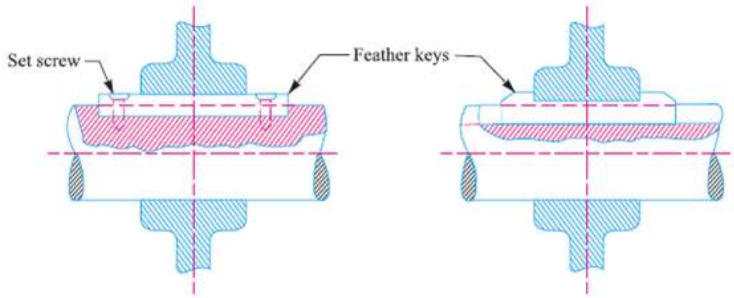


Figure 3 Feather Key

Woodruff key: Woodruff key is a sunk key in the form of a semi-circular disc of uniform thickness. Lower portion of the key fits into the circular keyway of the shaft. It can be used with tapered shafts as it can tilt and align itself on the shaft. But the extra depth of keyway in the shaft increases stress concentration and reduces strength of the shaft. Woodruff key is shown in Figure 4.

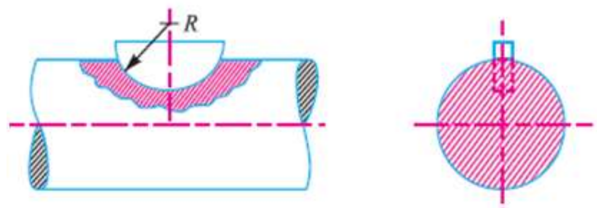
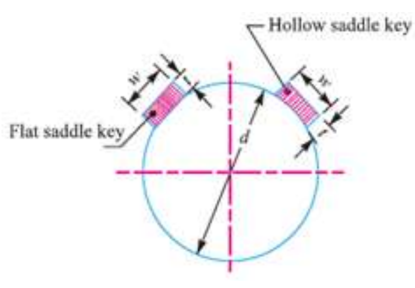


Figure 4 Woodruff Key

Saddle Keys

Slot for this type of is provided only in the hub as shown in Figure 5. Torque is transmitted by friction only and cannot therefore transmit high torque and is used only for light applications. The saddle keys are of two types: Flat Saddle Key and Hollow Saddle Key. In flat saddle key, the bottom surface touching the shaft is flat and it sits on the flat surface machined on the shaft. Hollow saddle key has a concave surface at the bottom to match the circular surface of the shaft. Chances of slip in case of the flat saddle key are relatively lesser and can transmit more power than the hollow saddle key.



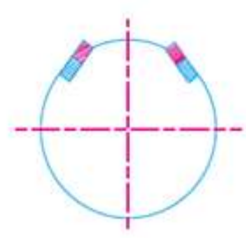


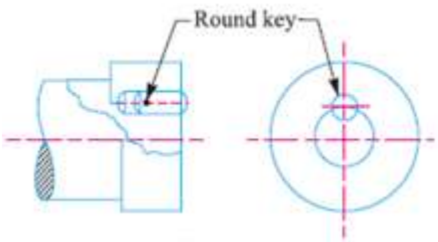
Figure 5 Saddle Keys Figure 6 Tangent Keys

Tangent Keys

Tangent keys are shown in Figure 6. These are used to transmit high torque. They may be used as a single key or a pair at right angles. Single tangent key can transmit torque only in one direction.

Round Keys

The round keys have a circular cross-section and fit into holes drilled partly in the shaft and partly in the hub. Slot is drilled after the assembly so the shafts can be properly aligned. These are used for low torque transmission. Round keys are shown in Figure 7.



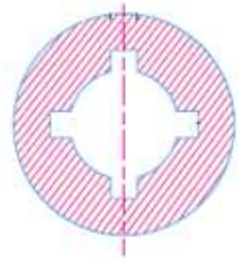


Figure7 Round Key Figure 8 Splines

Splines

A number of keys made as an integral part of the shaft are called splines. Keyways are provided in the hub. These are used for high torque transmission e.g. in automobile transmission. Splines also permit the axial movement. Splines are shown in Figure 8.

Design of Sunk Keys

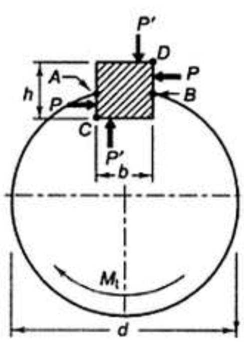
Figure 9 shows the forces acting on a rectangular key having width w and height h. Let l be the length of the key. Torque is transmitted from the shaft to the hub through key. Shaft applies a force P on the key and the key applies an equal force on the hub. Therefore the key is acted upon by two equal forces of magnitude P, one applied by the shaft (on the lower portion) and the other because of the reaction of hub (on the upper portion).

As these two forces are not in same plane, they constitute a couple which tries to tilt the key. Therefore equal and opposite forces P' also act on the key, which provide a resisting couple that keeps the key in position.

As the exact location of force P is not known, to simplify the analysis it is assumed that the force P acts tangential to the shaft. If T is the torque transmitted,

$$P = \frac{T}{d/2}$$

where, d = diameter of the shaft



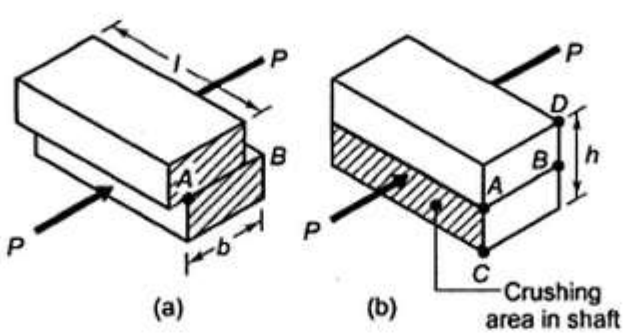


Figure 9 Forces Acting on Key Figure 10 Failure of Key a. Shear Failure b. Crushing Failure

In the design of key two types of failures are considered, shear failure and crushing failure.

Area resisting shear failure = w l

Shear stress, $\tau = \frac{P}{bl} \leq [\tau]$

Crushing Area = *l h/2*

Crushing stress, $\sigma_{crushing} = \frac{P}{l\ h/2} \leq [\sigma_c]$

Tables are available which give standard cross-sections for square and rectangular keys corresponding to different shaft diameters. But in the absence of such data, following relations are generally used:

For Rectangular Key: w = d / 4 and h = d / 6

For Square Key: w = h = d /4

For a known diameter of shaft, w and h can be calculated using these relations and then using the above strength equations required length of the key is calculated for given values of allowable stresses. Length is calculated both for shear and crushing and then maximum value out of the two is considered.

DESIGN OF PULLEYS

Introduction

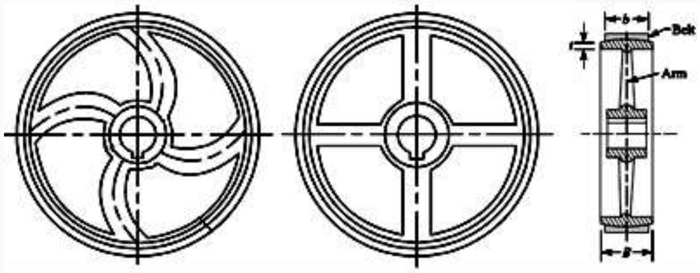
Pulleys are the wheels used to transmit power from one shaft to another, with the help of belts. Proper selection of diameters of pulleys is important as the velocity ratio is the inverse ratio of the diameters of driving and driven pulleys. For the belt to travel in a line normal to the pulley faces, the pulleys must be perfectly aligned with each other. Pulleys should have following important properties:

1. Ability to absorb shocks
2. High heat conductivity
3. High corrosion resistance
4. High coefficient of friction to reduce belt slippage
5. High strength to weight ratio

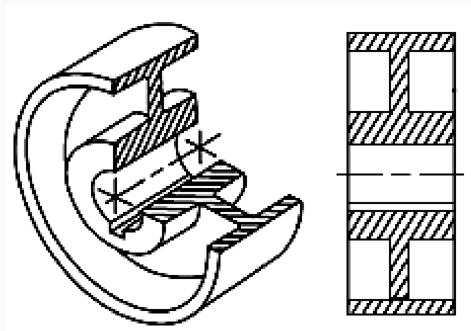
Pulley has three main components: i. Hub ii. Arms/spokes or web iii. Rim

Pulley Material

Pulleys are generally made of cast iron, forged steel, wood or compressed paper pulp. Because of their low cost, cast iron pulleys are most widely used. Arms or spokes of cast iron pulleys have elliptical cross-section and can have straight or curved shape as shown in figure: a. In some pulleys, instead of arms, web is provided to join hub with rim, as shown in figure : b. Split cast iron pulleys, shown in figure: 2, are made in two halves that are bolted together. These are easier to mount on shafts with a range of diameters, by tightening those on the shaft.



a. Pulleys with Arms



b. Pulley with Web

Figure:1 Solid Cast Iron Pulleys

Steel pulleys have higher strength, are lighter in weight and can run at higher speeds. But they have lower coefficient of friction in comparison to cast iron pulleys. Steel pulleys are generally made in two parts, which are bolted together on the shaft. Bushings are provided to take care of shafts of different diameters and for normal service; power is transmitted without key, only through frictional force obtained by clamping.

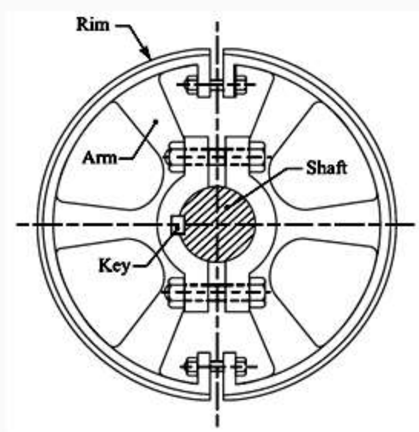
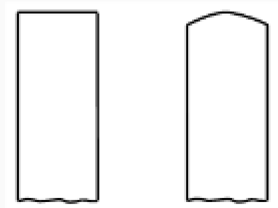


Figure:2 Split Cast Iron Pulley

Wooden pulleys are lighter and have higher coefficient of friction. Wooden pulleys are made of segments glued together under heavy pressure. Protective coatings of shellac or varnish are applied to avoid warping due to moisture. Wooden pulleys are also made as solid or split and have cast iron hubs with keyways. Adjustable bushings are also used in some of the wooden pulleys.

Paper pulleys are generally used for smaller centre distances to transmit power from electric motors. These are made from compressed paper fibre and are formed with a metal in the centre.

Crowning of Pulleys



a. Flat Pulley   b. Crowned Pulley

Figure: Crowning of Pulleys

In case of flat belts, thickness of the rim is increased from the centre so that it gets convex shape as shown in figure 21.3. This is known as crowning of the pulley. It helps in preventing the belt from running off the pulley by bringing it to the mid-plane of pulley whenever it moves to sides. Thus it helps in keeping the belt running in equilibrium position near the centre of the rim.

Design of Cast Iron Pulleys

The following procedure may be adopted for the design of cast iron pulleys.

Dimensions of Pulley

a. Diameter of the pulley

Diameter of the pulley (D) is selected depending upon the required velocity ratio or centrifugal stress.

The centrifugal stress obtained in the rim of the pulley,

$\sigma_T = \rho V^2$……… where ρ is the density of the rim material and V is the velocity of the rim. $V = \frac{\pi DN}{60}$

b. Width of the pulley(B)

Width of the pulley or face of the pulley (B) is taken 1.25 times the width of belt (b).

B = 1.25 b

c. Thickness of the pulley rim (t)

Thickness of the pulley rim (t) is taken between (D/300 +2)mm and (D/300 +3)mm for single belt and for double belt.

d. Dimensions of Arms

Number of arms can be taken as follows:

Pulley Diameter	**No. of Arms**
< 200 mm	Web (with thickness = rim thickness)
> 200mm and < 450mm	4
> 500 mm	6

Cross-section of arms is generally elliptical with major axis (a_1) equal to twice the minor axis (b_1). The cross-section of the arm is obtained by considering the arm as cantilever, fixed at the hub end and carrying a concentrated load at the rim end and having a length equal to the radius of the pulley. Also, it is assumed that at any given time, the power is transmitted from the hub to the rim or vice versa, through only half of the total number of arms.

Tangential load per arm is given by,

$$F_t = \frac{T}{R \times \frac{n}{2}} = \frac{2T}{R\,n}$$

where, T = Torque transmitted

R = Radius of pulley

n = Number of arms

Maximum bending moment on the arm at the hub end,

$$M = F_t \times R = \frac{2T}{n}$$

Maximum bending stress is given by,

$$\sigma_b = \frac{M\,y}{I} \leq [\sigma_t]$$

where, I = Moment of inertia of cross-sectional area of the arm about the axis of rotation

$$I = \frac{\pi b_1 a_1^3}{64} = \frac{\pi b_1^4}{8} \quad \text{as } a_1 = 2b_1$$

y = Distance from neutral axis to the outer most fibre = $a_1 / 2 = b_1$

A taper, generally of 1/48 to 1/32, is provided on the arms, from hub to rim.

e. **Dimensions of Hub**

For known shaft diameter (d), diameter of the hub (d_1) can be taken as:

$$d_1 = 1.5d + 25\ mm$$

But the diameter of the hub should not be greater than 2d.

Length of the hub can be taken as,

$$L = \frac{\pi}{2} \times d$$

The minimum length of the hub is $2/3^{rd}$ of the width of the pulley and it should not be more than the width of the pulley.

DESIGN OF COUPLINGS

1.1 Introduction

Couplings are used to connect two rotating shafts to transmit torque from one to the other. For example coupling is used to connect the output shaft of an electric motor to the input shaft of a hydraulic pump.

1.2 Types of Shafts Couplings

Rigid Couplings

Rigid Couplings are used to connect two shafts which are perfectly aligned. These are simple and inexpensive.

Rigid Couplings are of following types:

- Sleeve or Muff Coupling
- Clamp or Split-muff or Compression Coupling
- Flange Coupling

Flexible couplings:

Flexible couplings are used to connect two shafts having lateral or angular misalignment. Flexible elements provided in flexible coupling absorb shocks and vibrations.

Flexible Couplings are of following types:

- Bushed pin type Coupling
- Universal Coupling
- Oldham coupling

Flange Coupling

Introduction

Flange coupling consists of two flanges keyed to the shafts. The flanges are connected together by means of bolts arranged on a circle concentric to shaft. Power is transmitted from driving shaft to flange on driving shaft through key, from flange on driving shaft to the flange on driven shaft through bolts and then to the driven shaft through key again. Projection is provided on one of the flanges and a corresponding recess is provided in the other for proper alignment. Flange coupling is of two types – unprotected and protected. If in case failure of bolts occurs during the operation, the bolts may hit the operator in case of unprotected flange coupling. To avoid this, protective circumferential flanges are provided in the protected type flange coupling.

Flange of a protected type flange coupling has three distinct regions – inner hub, flanges and protective circumferential flanges. Following standard proportions are used in the design of flange coupling:

Outer diameter of hub, $D = 2\ d$

Pitch circle diameter of bolts, $D_1 = 3\ d$

Outer diameter of flange, $D_2 = 4\ d$

Length of the hub, $L = 1.5\ d$

Thickness of flange, $t_f = 0.5\ d$

Thickness of protective circumferential flange, $t_p = 0.25\ d$

where d is the diameter of shafts to be coupled.

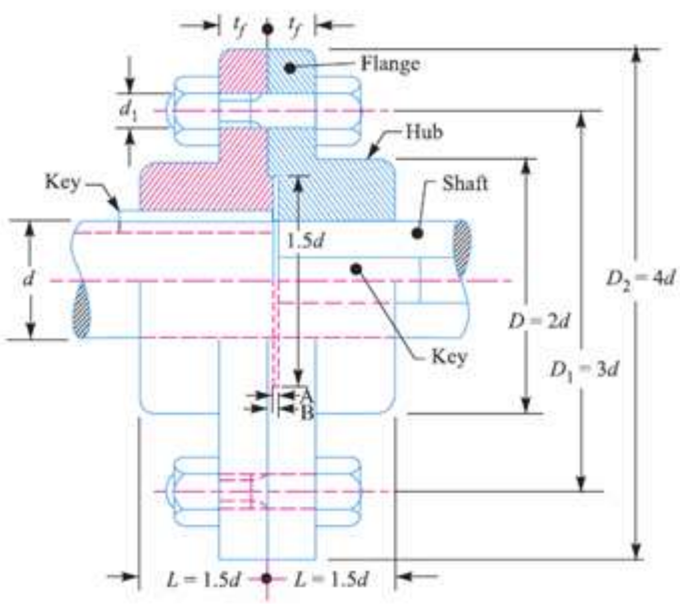


Figure 1.4 Flange Coupling

1.5.2 **Design**

1.5.2.1 <u>Design of Shafts</u>

Shafts are designed on the basis of torsional shear stress induced because of the torque to be transmitted. Shear stress induced in shaft for transmitting torque, T is given by,

$$\tau = \frac{T\ r}{J} \leq [\tau]$$

Where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation

r = Distance from neutral axis to the outer most fibre = d/2

So dimensions of the shaft can be determined from above relation for a known value of allowable shear stress, [τ].

1.5.2.2 Design of Hub

Hub is designed considering it as a hollow shaft, with inner diameter equal to diameter of shafts and outer diameter double of that. It is checked for torsional shear stress.

Shear stress, $$\tau = \frac{T\,r}{J} \leq [\tau]$$

Where T = Twisting moment (or torque) to be transmitted

J = Polar moment of inertia about the axis of rotation

r = Distance from neutral axis to the outer most fibre = D/2

1.5.2.3 Design of Key

In this case two separate keys are used for the two shafts. Key is designed as discussed earlier. In this case, length of key, (length of the hub)

1.5.2.4 Design of Flange

The flange is subjected to shear at the junction of the hub as it transmits torque through the bolts. Area resisting shear

where, t_f is the thickness of the flange.

If T is the torque to be transmitted, tangential force, $$F = \frac{T}{d/2}$$

Shear stress, $$\tau = \frac{F}{\pi D t_f} \leq [\tau]$$

1.5.2.5 Design of Bolts

Due to transmission of torque, force acts perpendicular to the bolt axes and the bolts are subjected to shear and crushing stresses. Let n be the total number of bolts.

Force acting on each bolt, $$F_b = \frac{T}{n\,D_1/2}$$

where D_1 is the pitch circle diameter of bolts.

Area resisting shear $$= \frac{\pi}{4} d_c^2$$

where, d_c = core diameter of bolts

Shear stress, $$\tau = \frac{F_b}{\frac{\pi}{4} d_c^2} \leq [\tau]$$

Area under crushing

Crushing stress, $\sigma_{crushing} = \frac{F_b}{d_c . t_f} \leq [\sigma_c]$

Pbm1. Two shafts are connected by a C.I flange coupling which transmits 40kW at 350rpm. Assume for shaft, key and bolt material f_t = 45 N/mm^2, f_S = 40 N/mm^2 , f_C = 60 N/mm^2 and f_S for CI as15 N/mm^2. Consider all the possible failure for the different parts

i) Design of shafts
ii) Design of bolts
iii) Design the key

Pbm2. Design a flange coupling of protected type of mild steel shaft which is to transmit 59KW at 240 rpm. The maximum torque is 30% greater than mean torque. The allowable shear stress in CI hub is 6 MPa and allowable shear stress in shaft and key is 40MPa. The allowable shear stress on bolt is 30MPa. Crushing stress in key is 125MPa.

Solution:

Given data:

P = 59kW

N = 240 rpm

τ_{SH} = 6 MPa hub

τ_{SS} = 40 MPa shaft

τ_{SK} = 40 MPa key

τ_{SB} = 30 MPa bolt

τ_{CK} = 125 MPa

Crushing key

Step1:

Torque submitted by the shaft $T = \frac{P\ X\ 60}{2\pi N}$ = 2347.53Nm

Design torque $T_D = 1.3 \ X\ T$ = 3052 Nm 3052X10^3 Nmm

$T_D = \frac{\pi \tau_{SS} d^3}{16}$ = d = 72.97mm = 75mm

Step2: empirical relation:

Outer diameter of hub, D = 2 d = 150mm

Length of the hub, L = 1.5 d = 112.5mm = 115mm

Step3:

Design of Hub: $T_{D\,i} = \frac{\pi}{16} . \left(\frac{D^4 - d^4}{D}\right) . \tau_{SH}$

Find $\tau_{SH\ induced}$ = 4.91 Mpa

Design is safe

Step4:

Design of key:

Assume width of the key, w $= \frac{1}{3}$rd of d = thickness of key, t = 25mm

length of the key = length of the hub = l =115mm

torque induced in key due to shearing, $T_D = l \times w \times \frac{d}{2} \times \tau_{SK\ induced}$

$\tau_{SK\ induced}$ = 28.31 N/mm^2 = 28.31MPa

torque induced in key due to crushing, $T_D = l \times \frac{t}{2} \times \frac{d}{2} \times \tau_{CK\ induced}$ $\tau_{CK\ induced}$ = 56.62 MPa

design is safe

step 5:

Design of flange:

Outer diameter of flange, $D_2 = 4\ d$ = 300mm

Thickness of flange, $t_f = 0.5\ d$ = 40mm

Thickness of protective circumferential flange, $t_p = 0.25\ d$ = 20mm = 15mm

If T is the torque to be transmitted, tangential force $F = \frac{T_D}{\frac{d}{2}}$ = 81386.67 N

Shear stress in flange $= \frac{F}{\pi D t_f}$ = 4.32 N/mm^2 $\leq$ τ_{SH}

design is safe

Step 6:

Design of bolts:

Pitch circle diameter of bolts, $D_1 = 3\ d = 225mm$

Force acting on each bolt, $F_B = \frac{2XT_D}{n.D_1} = \frac{2X3052x10^3}{4\ X225} = 6782.23N$

Area resisting shear, $A = \frac{\pi}{4} d_C^2 = 2827.4mm^2$ d_C core diameter take $d_C = 0.8\ Xd = 60mm$

Shear stress $= \frac{F_B}{A} = 2.39\ N/mm^2 \leq \tau_{SB}$

Crushing stress, σ_{SB} induced $= \frac{F_B}{d_C X t_F} = 2.825\ N/mm^2 \leq \sigma_{SB}$

Remember if d<40mm take n = 3

If 40>d<200 take n = 4

If d>200 take n = 6