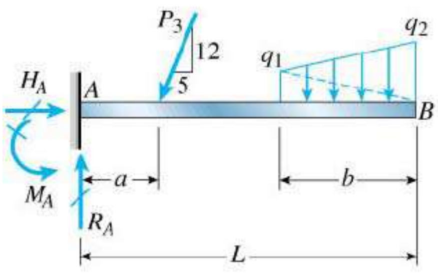
Chapter: 2
Shear force and bending moment:

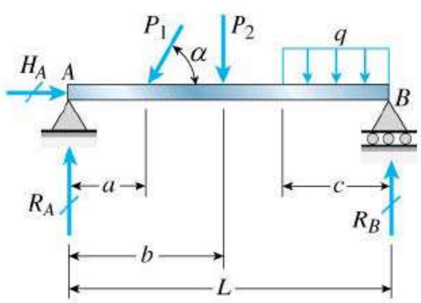
Beam: Any member of a machine or structure whose one dimension (length) is very large as compared to the other two dimensions (width and thickness) and which can carry lateral or transverse loads in the axial plane is called a beam. A beam may be of rectangular, square, triangular, hexagonal and circular, etc. cross-sections. A beam is a very important member in structural mechanics to withstand the transverse loads. A beam may be made '-of timber, flitched beam, i.e. timber reinforced with mild steel strips, steel, and reinforced concrete. The reinforced concrete beams are mostly used in building construction, bridges and flyovers.

Types of Beams:

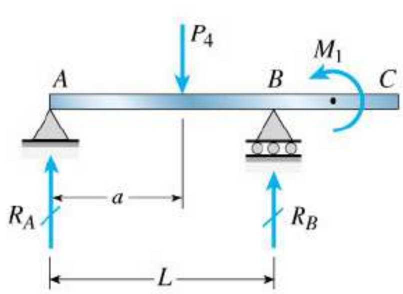
i) *Cantilever beam*: A cantilever beam is a beam whose one end is fixed and the other end is free.



ii) *Simply supported beam*: A simply supported beam is one which has hinge at one end and roller support at the other, the ends are freely rests on walls or columns or knife edges.

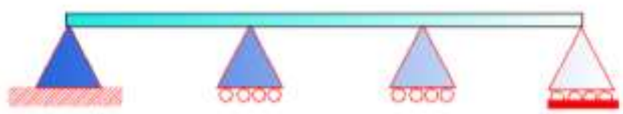


iii) *Over hanging beam*: an overhanging beam is one in which the supports are not situated at the ends.



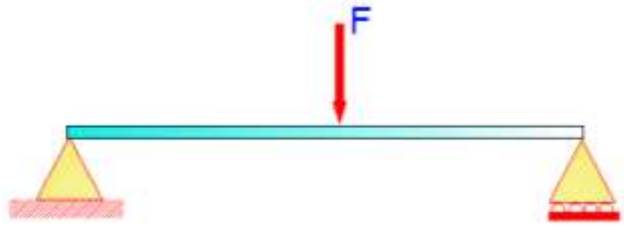
iv) *Rigidly fixed or built in beam*: A fixed beam is one whose both ends are rigidly fixed or built in into its supporting walls or columns.

v) *Continuous beam*: A continuous beam is one which has more than two supports. The supports at the extreme left and right are called end supports and all the other supports, except the extreme are called intermediate support.

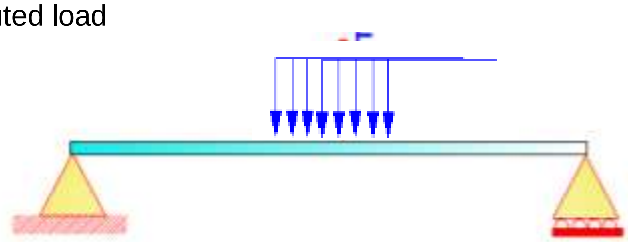


Types of Loading:

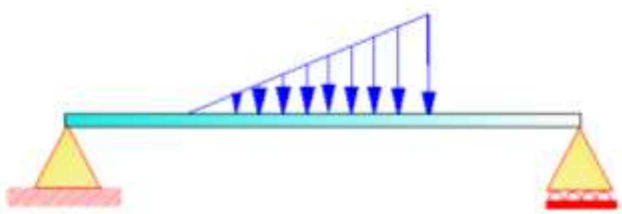
i) concentrated or point load



ii) Uniformly distributed load



iii) Uniformly varying load



Bending moment: The bending moment (B.M.) at any point along a loaded beam is the algebraic sum of the moments of all the vertical forces acting to one side of the point about the point. Consider a simply supported beam AB carrying concentrated loads as shown in Figure below. Let R_A and R_B be the vertical reactions at supports A and B respectively. Consider a section xx at a distance x from end A. The clockwise moment at this section due to all the loads acting on the beam to the left of the section is:

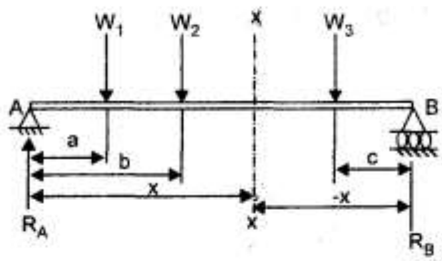


Fig. 3.1

$$M_x = R_A x - W_1 (x - a) - W_2 (x - b)$$

If we consider the forces to the right of the section x—x, then anticloc4wise moment is:

$$M_x = R_B (l - x) - W_3 (l - x - c)$$

For equilibrium of the beam, the B.M. given by equations (1) and (2) are equal. The SI units of B.M. are N.m or kN.m.

Shear force: The shear force (S F) at any point along a loaded beam is the algebraic sum of all the vertical forces acting to one side of the point. Thus, for the beam AB shown in Figure, the shear force at cross-section x—x as measured from the left hand side is:

$$F_x = R_A - W_1 - W_2 \text{ ------(1)}$$

The shear force as measured from the right hand side is:

$$F_x = W_3 - R_B \text{ ----------(2)}$$

Since the beam is in equilibrium, the S.F. given by equations (1) and (2) are equal.

Sign conventions

A B.M is considered positive if it produces compression on the top fibers of the beam and negative if it produces tension on the top fibers of the beam. Positive B.M. is called sagging moment and negative B.M. as hogging moment.

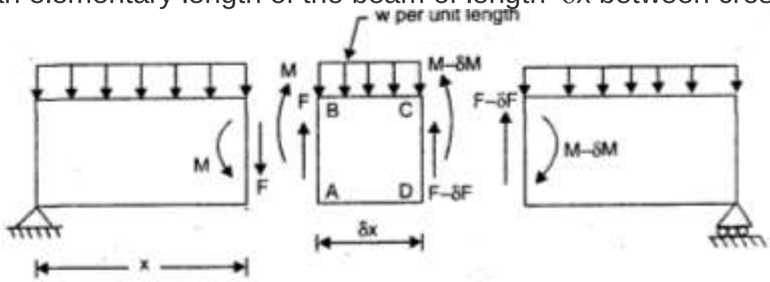
(a) +ve or sagging B.M. (b) –ve or hogging B.M.

The shear force is positive if it tends to move left position upward relative to right portion.

(a) +ve S.F. (b) –ve S.F.

RELATIONSHIP BETWEEN LOAD, SHEAR FORCE AND BENDING MOMENT:

Consider a simply supported beam carrying a UDL of intensity w per unit length as shown in Figure. Consider an elementary length of the beam of length δx between cross-Sections



x and $x+\delta x$. Let the bending moment at x be M and $x+\delta x$ it be $M-\delta M$. The shear force at x is F and at $x+\delta x$ it is $F-\delta F$. For the equilibrium of the beam element ABCD, we have

$$\Sigma F_y = 0; \quad F-(F-\delta F)-w.\delta x = 0$$

$$\delta F - w.\delta x = 0 \quad \text{or} \quad \frac{\delta F}{\delta x} = w$$

In the limit as $\delta x \to 0$, we get

$$\frac{dF}{dx} = w$$

Therefore, the first derivative of shear force with respect to x at a point gives the intensity of loading at the point.

$$\Sigma M_A = 0; \ (F-\delta F).\delta x-(M-\delta M)+w.\delta x.\frac{\delta x}{2}+M = 0$$

$$F.\delta x-\delta F.\delta x - M + \delta M + w.\frac{\delta x^2}{2}+M = 0$$

Neglecting small quantities, we get

$$F.\delta x + \delta M = 0$$

$$\frac{\delta M}{\delta x} = -F$$

In the limit as $\delta x \to 0$, we get

$$\frac{dM}{dx} = -F$$

Therefore, the rate of change of bending moment with respect to x is equal to the shear force. Whenever, bending moment is maximum or minimum, the shear force is zero.

Taking the derivative again, we get,

$$\frac{d^2M}{dx^2} = -\frac{dF}{dx} = -w$$

B.M. AND S.F. DIAGRAMS

B.M. Diagram:

To draw the B.M.D, the following procedure may be followed:

1. Take a sheet of graph paper. Draw the beam along with loading to an appropriate scale.
2. Calculate the reactions at the supports by applying the equations of equilibrium
3. Choose a section x-x at a distance x from the left hand support. The section may be chosen either after every concentrated load or before the right hand support. For UDL, the section may be taken within the load.
4. Calculate the B.M. beneath every concentrated load. For UDL, the B.M. may be calculated along the length of the load. Ignore that term which becomes negative on substituting the value of x.

. 5. Draw the B.M.D. for the beam on a convenient scale. Of course, sign convention has to be followed for B.M.

Point of Inflexion. It is the point on the beam in the B.M.P. where the bending moment becomes zero.

Point of Contra flexure. It is the point in the B.M.D where the B.M. changes slope from an increasing one to a decreasing one. Contra flexure means opposite and flexure means bending. Some authors consider point of inflexion and point of contra flexure to be synonymous.

S.F. Diagram:

The first three steps for the S.F.D. are the same as for the B.M.D., and need not be repeated.

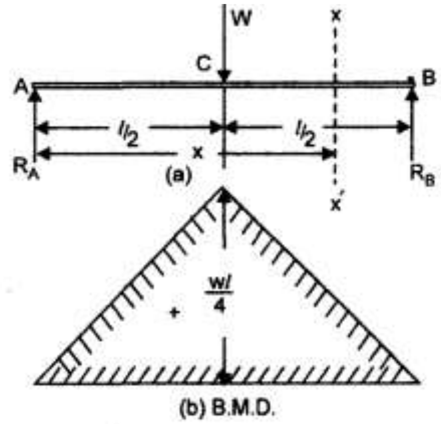
4. Calculate the S.F. beneath every concentrated load just to the left and to the right.

For UDL, the S.F. has to be calculated along the length of the load.

5. Draw the S.F.D. to a convenient scale using the sign convention.

B.M AND S.F DIAGRAMS FOR A SIMPLY SUPPORTED BEAM

.1 Concentrated Load at Midspan



Consider a beam AB simply supported at the ends and carrying a concentrated load W at midspan C as shown in Fig. 3.4 (*a*).

$\Sigma F_y = 0$; $\quad R_A + R_B = W$

$\Sigma M_B = 0$; $\quad R_A \times l = W \times \frac{l}{2}$

$$R_A = \frac{W}{2}$$

$$R_B = \frac{W}{2}$$

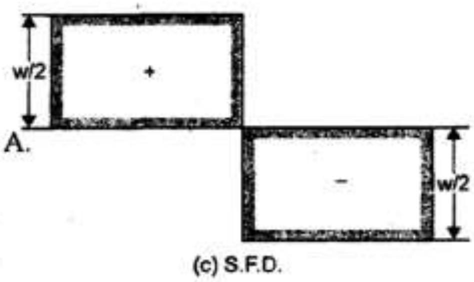


Fig. 3.4

B.M.D. Take a section x–x at a distance x from A.

$$M_x = R_A x - W\left(x - \frac{l}{2}\right)$$

$$= \frac{W}{2}x - W\left(x - \frac{l}{2}\right)$$

This represents a straight line.

x	0	$\frac{l}{2}$	l
M_x	0	$\frac{Wl}{4}$	0

The B.M.D. is shown in Fig. 3.4 (*b*)

$$M_{max} = \frac{Wl}{4} \text{ and occurs at the mid-span.}$$

S.F.D.

Between AB, $\quad$ S.F. $= \frac{W}{2} \uparrow$ and positive in nature.

Between BC, $\quad$ S.F. $= \frac{W}{2} - W = -\frac{W}{2} \downarrow$, *i.e.* S.F. is negative in nature.

The S.F.D is shown in Fig. 3.4 (*c*).

Uniformly Distributed Load

Consider a beam AB of span 1 simply supported at the ends and carrying a uniformly distributed load of intensity w per unit length as shown in Fig. 3.5 (a)

$\Sigma F_y = 0; \quad R_A + R_B = wl$

$\Sigma M_B = 0; \quad R_A \times l = wl \times \frac{l}{2}$

$$R_A = \frac{wl}{2}$$

$$R_B = \frac{wl}{2}.$$

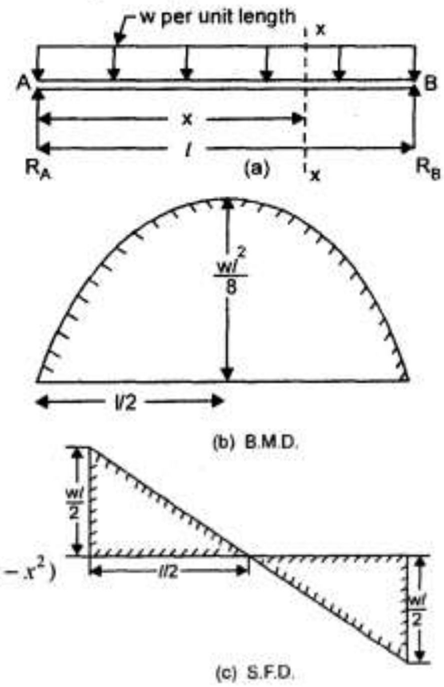


Fig. 3.5

B.M.D.

Consider a section x–x at a distance x from A.

$$M_x = R_A x - wx \times \frac{x}{2}$$

$$= \frac{w}{2} lx - \frac{wx^2}{2} = \frac{w}{2}(lx - x^2)$$

This represents a parabola

x	0	$\frac{l}{2}$	l
M_x	0	$\frac{wl^2}{8}$	0

This B.M.D. is shown in Fig. 3.5 (*b*).

S.F.D

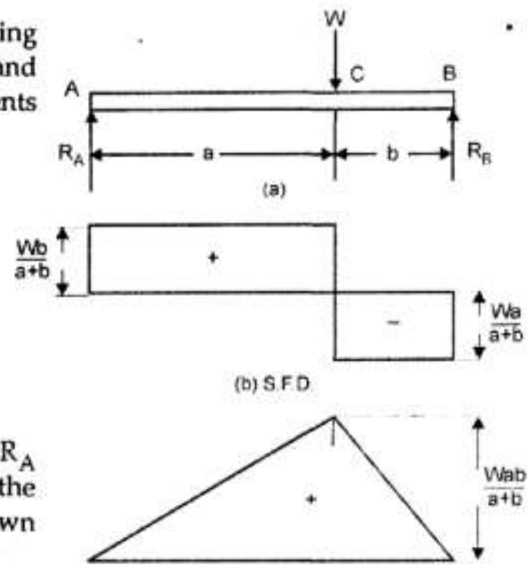
$$F_x = R_A - wx = \frac{wl}{2} - wx = w\left(\frac{l}{2} - x\right)$$

This represents a straight line.

x	0	$\frac{l}{2}$	l
F_x	$\frac{wl}{2}$	0	$\frac{-wl}{2}$

The S.F.D. is shown in Fig. 3.5 (*c*).

Concentrated load at mid span:



(c) B.M.D.

Fig. 3.7

Consider a simply supported beam carrying concentrated load W at a distances '*a*' from left hand support, as shown in Fig. 3.7 (*a*). Taking moments about end B, we have

$$R_A\,(a+b) = Wb$$

$$R_A = \frac{Wb}{a+b}$$

and $$R_B = \frac{Wa}{a+b}$$

S.F.D.

Between span AC, the shear force is R_A upwards and is positive. Between span CB, the shear force is R_B and is negative. The S.F.D. is shown in Fig. 3.7 (*b*)

B.M.D.

At a distance x from A, the bending moment is,

$$M_x = R_A \cdot x = \left(\frac{Wb}{a+b}\right)x$$

This represents a sbraight line.

At $x = a$

$$M_C = \frac{Wab}{a+b}$$

This is a positive bending moment.

Between span CB,

$$M_x = R_A x - W\,(x-a)$$

$$= \left(\frac{Wb}{a+b}\right) x - W\,(x-a)$$

$$= Wa - \left(\frac{Wa}{a+b}\right)x$$

This also represents a straight line.

At $x = a,\ M_C = \frac{Wab}{a+b}$

At $x = a + b$, $M_B = 0$

The B.M.D. is shown in Fig. 3.7 (*c*).

CANTILEVER BEAM

Concentrated Load at Free End

Consider a cantilever beam AB of span 1 carrying a concentrated load P at the free end as shown in Fig. 3.9 (*a*). Consider a section *x*–*x* at distance *x* from the free end.

B.M.D.

$$M_x = Px$$

It is a negative B.M. as it produces tension on the top fibres of cantilever. It represents a straight line.

$$M_{max} = Pl$$

The B.M.D. is shown in Fig. 3.9 (*b*).

S.F.D.

The shear force at section *x*–*x* is :

$$F_x = P$$

It is a positive shear force and is constant over the whole length. The S.F.D. is shown in Fig. 3.9 (*c*).

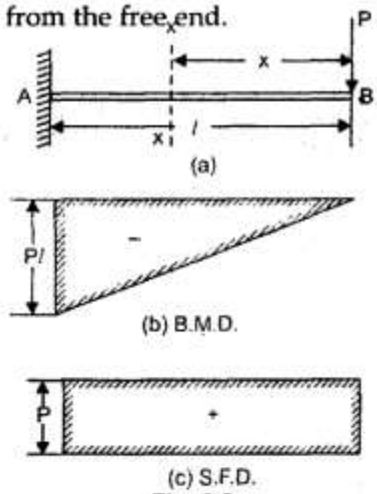


Fig. 3.9

Uniformly Distributed Load over Whole Span

Consider a cantilever beam AB of span *l* carrying a *udl* of intensity ω over the whole span as shown in Fig. 3.10 (*a*). Consider a section *x*–*x* at a distance *x* from free end.

B.M.D.

$$M_x = wx \cdot \frac{x}{2} = \frac{wx^2}{2}$$

$$M_{max} = \frac{wl^2}{2} \text{ at } x = l$$

$$\text{B.M.} = 0 \text{ at } x = 0$$

It represents a parabola. The B.M.D. is shown in Fig. 3.10 (*b*).

S.F.D. :

$$F_x = wx$$

At $x = 0$, $F = 0$

and at $x = l$, $F = wl$

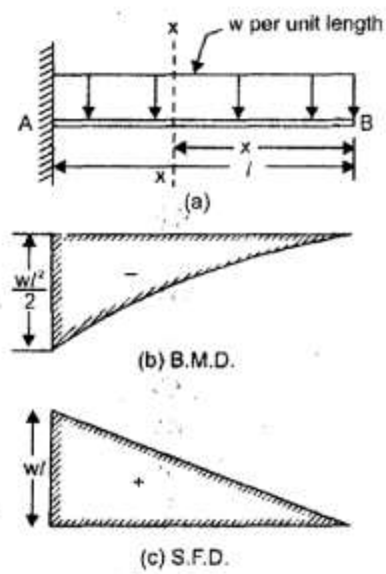


Fig. 3.10

It represents a straight line giving linear variation. The S.F.D is shown in Fig. 3.10 (c).

OVERHANGING BEAMS

Concentrated Loads

Consider a beam AB of span l having overhangs CA and BD, each equal to b. It carries concentrated loads W_1 to W_4 as shown in Fig. 3.12.

$$\Sigma F_y = 0;$$

$$R_A + R_B = W_1 + W_2 + W_3 + W_4$$

$$\Sigma M_B = 0;$$

$$R_A \times l + W_4 \times b = W_1 (l + b) + W_2 (l - a) + W_3 a$$

$$R_A = W_1 \left(\frac{l+b}{2}\right) + W_2 \left(\frac{l-a}{l}\right) + W_3 \frac{a}{l} - W_4 \frac{b}{l}$$

or $$R_A = W_1 \left(1 + \frac{b}{l}\right) + W_2 \left(1 - \frac{a}{l}\right) + W_3 \frac{a}{l} - W_4 \frac{b}{l}$$

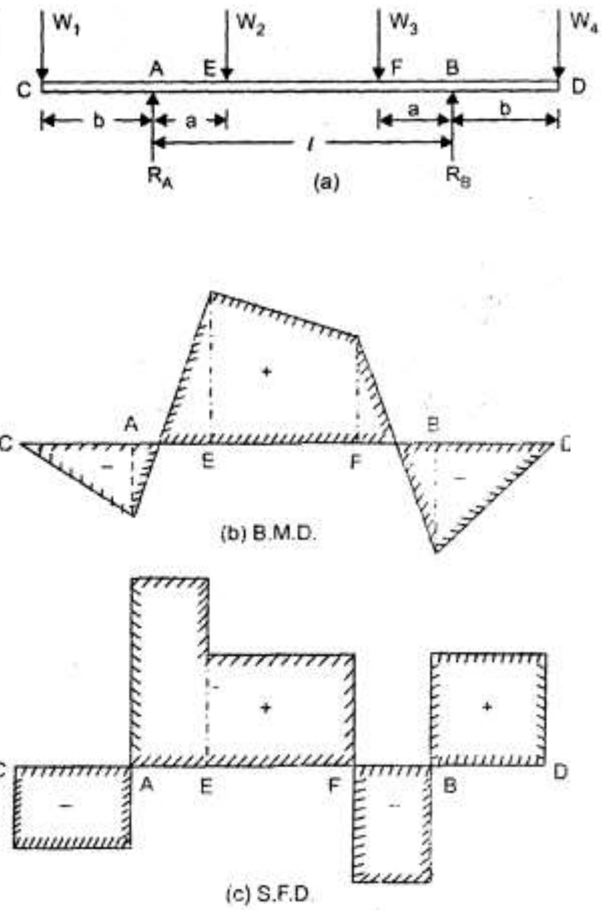


Fig. 3.12

B.M.D. :

Span CA : At a distance x from C,

$$M_x = -W_1 x$$

At $x = 0$, $M = 0$; $x = b$, $M_A = -W_1 b$

This is a negative B.M. as it produces tension on the top fibres. It is a linearly varying B.M.

Span AB : At a distance x from C near to B,

$$M_x = -W_1 x + R_A (x - b) - W_2 (x - a - b) - W_3 (x - l + a - b)$$

It gives linear variation of B.M. The B.M. will change over from negative to positive value depending upon the value of loads.

Span BD : $M_B = -W_4 b$

The B.M.D. is shown in Fig. 3.12 (b).

S.F.D :

Span CA : S.F. in CA $= -W_1$

S.F. at A $= R_A - W_1$

Span AB : At E $= -W_1 + R_A - W_2$

At F $= -W_1 + R_A - W_2 - W_3$

Span BD : S.F. in DB $= +W_4$

At B $= R_B - W_4$

The S.F.D. is shown in Fig. 3.12 (c).

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Uniformly Distributed Load

Consider a beam ABCD (Fig. 3.13*a*) of span l and having overhangs equal to 'a' on both sides and subjected to a *udl* of intensity w per unit length.

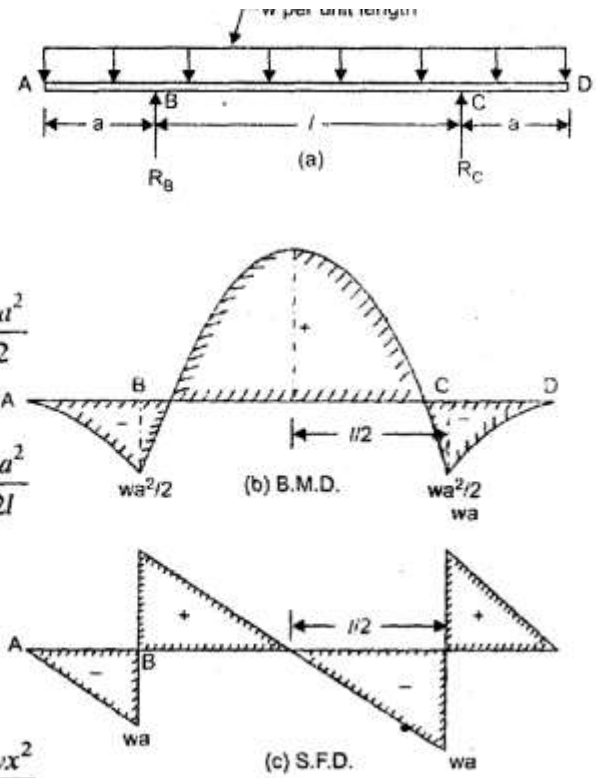


Fig. 3.13

$\Sigma F_y = 0;\qquad R_B + R_C = w(l + 2a)$

$\Sigma M_C = 0;\qquad R_B \times l = \dfrac{w(a+l)^2}{2} - \dfrac{wa^2}{2}$

$$R_A = R_B$$

$$= \frac{w(a+l)^2}{2l} - \frac{wa^2}{2l}$$

$$= \frac{w(l+2a)}{2}$$

B.M.D. :

Span AB : At a distance x from A,

$$M_x = -wx \times \frac{x}{2} = -\frac{wx^2}{2}$$

At A, $x = 0$, $M_A = 0$

At B, $x = a$, $M_B = -\dfrac{wa^2}{2}$

The variation of B.M. is parabolic.

Span BC :

$$M_x = -\frac{wx^2}{2} + R_B(x-a) = -\frac{wx^2}{2} + \frac{w(l+2a)}{2}(x-a)$$

The variation of B.M. is parabolic.

At $x = a + \dfrac{l}{2}$, $\qquad M = \omega\left(\dfrac{l^2}{8} - \dfrac{a^2}{2}\right)$

Span CD :

At a distance x from D,

$$M_x = -\frac{wx^2}{2}$$

The variation of B.M. is parabolic.

At D, $x = 0$, $\qquad M_D = 0$

At C, $x = a$, $\qquad M_C = -\dfrac{wa^2}{2}$

The B.M.D. is shown in Fig. 3.13 (*b*).

S.F.D.

At a distance x from A.

Span AB : $F_x = -wx$

At $x = 0$, $F_A = 0$

$x = a$, $F_B = -wa$

Span BC :

$$F_x = -wx + R_B = -wx + \frac{w(l+a)}{2}$$

Span CD :

At a distance x from D,

$$F_x = wx$$

$$F_D = 0, F_C = wa$$

The variation of shear forces is linear. The S.F.D. is shown in Fig. 3.13 (*c*).

Example1: Draw the bending moment and shear force diagrams for the simply supported beam loaded as shown in Fig. 3.16(a).

Solution. $\Sigma F_y = 0$:

$$R_A + R_B = 5 + 6 + 4 = 15 \text{ kN}$$

$\Sigma M_B = 0$;

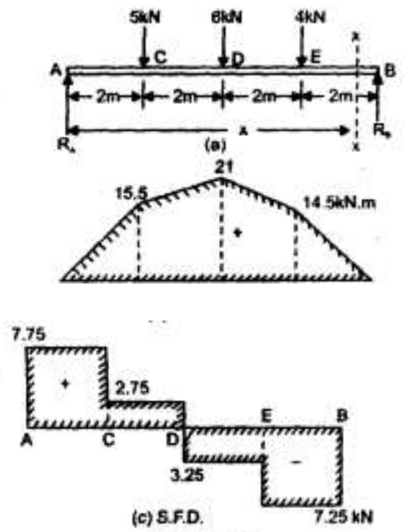
$$R_A \times 8 = 5 \times 6 + 6 \times 4 + 4 \times 2$$
$$= 62$$
$$R_A = 7.75 \text{ kN}$$
$$R_B = 7.25 \text{ kN}$$

Consider a cross-section x–x of the beam near to the support B at a distance x from A.

B.M.D.

$$M_x = R_A \times x - 5(x-2) - 6(x-4) - 4(x-6)$$
$$= 7{\cdot}75x - 5(x-2) - 6(x-4) - 4(x-6)$$

The variation of B.M. is linear throughout. A bracket is ignored, if on substituting the



x, m	0	2	4	6	8
M, kNm	0	15·50	21·0	14·5	0

The B.M.D. is shown in Fig. 3.16 (*b*) to a scale of 1 mm = 1 kN.m.

Fig. 3.16

S.F.D.

Point	A	–	C	–	D	–	E	–	B
Span	–	AC	–	CD	–	DE	–	EB	–
F_{kN}	7·75	7·75	2·75	2·75	–3·25	–3·25	–7·25	–7·25	–7·25

The S.F.D. is shown in Fig. 3.16(*c*) to a scale of 1 mm = 0·5 kN

Example .2 Draw the bending moment and shear force diagrams for the simply supported beam shown in Fig. 3.17(a).

Solution. $\Sigma F_y = 0$;

$$R_A + R_B = 5 + 2 + 2 + 10 = 19 \text{ kN}$$

$\Sigma M_B = 0$:

$$R_A \times 6 = 5 \times 5 + 2 \times 2 \times 3 + 10 \times 1 = 47$$

$$R_A = 7.833 \text{ kN}$$

$$R_B = 11.167 \text{ kN}$$

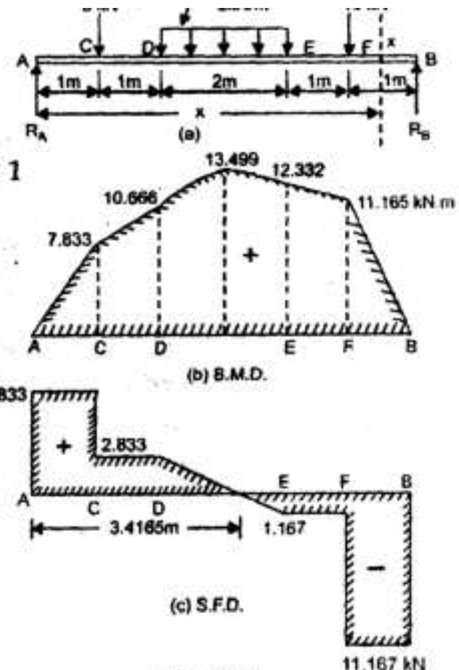


Fig. 3.17

B.M.D.

Taking a section x–x at a distance x from A, we have

$$M_x = R_A \times x - 5(x-1) - 2 \times 2 \times (x-3) - 10(x-5)$$

$$= 7.833\,x - 5(x-1) - 4(x-3) - 10(x-5)$$

x, m	0	1	2	3	4	5	6
M, kN.m	0	7.833	10.666	13.4999	12.332	11.165	0

The B.M.D. is shown in Fig. 3.17 (*b*) to a scale of 1 mm = 0.5 kN.m.

The B.M. curve between span DE is a parabola. In the remaining span it is linear.

S.F.D. S.F. at A, F_A = 7.833 kN

S.F. between span AC = 7.833 kN

S.F. at C, F_C = 7.833 − 5 = 2.833 kN

S.F. between spand CD = 2.833 kN

F_D = 2.833 kN

S.F. between span DE = 2.833 − 2 (x − 2)

At x = 3m, S.F. = 2.833 − 2 × 1 = 0.833 kN

At x = 4m, S.F., F_E = − 1.167 kN

S.F. between span EF = − 1.167 kN

F_F = − 1.167 − 10 = − 11.167 kN

S.F. between span FB = − 11.167 kN

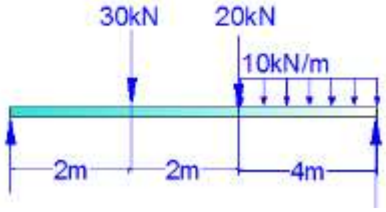
F_B = − 11.167 kN

The S.F.D. is shown in Fig. 3.17(*c*) to a scale of 1 mm = 0.5 kN.

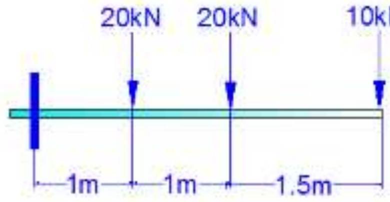
For S.F. to be zero between DE,

$$2.833 - 2(x-2) = 0 ; \quad x = 3.4165 \text{ m.}$$

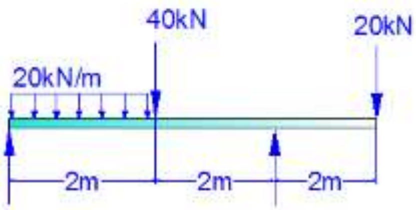
Pbm1. The simply supported beam shown in figure carries two concentrated loads and a uniformly distributed load. Draw the SFD and BMD.



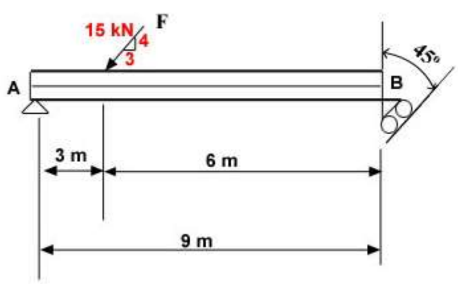
Pbm2. Draw the BMD and SFD for the cantilever beam as shown in figure.



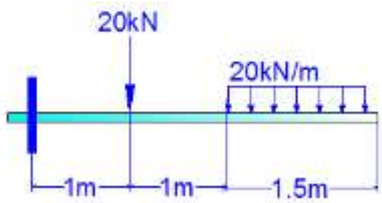
Pbm3. The simply supported beam shown in figure carries two concentrated loads and a uniformly distributed load. Draw the SFD and BMD.



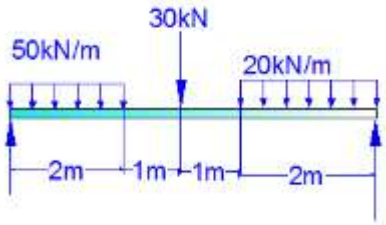
Pbm4. Determine the reactions at A and B for the beam shown due to the applied force.



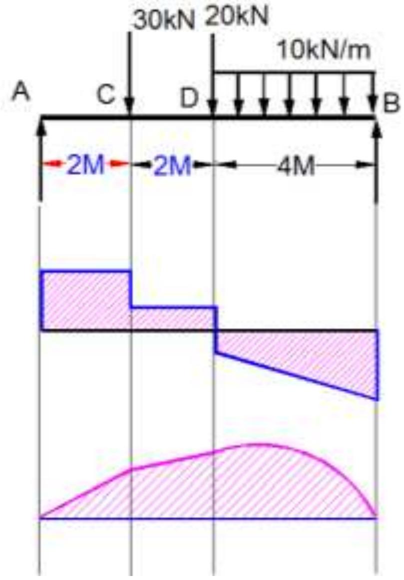
Pbm5. Draw the BMD and SFD for the cantilever beam as shown in figure.



Pbm6. Draw the BMD and SFD for the beam as shown in figure.



Pbm1. The simply supported beam shown in figure carries two concentrated loads and a uniformly distributed load. Draw the SFD and BMD.



Solution: $\Sigma M_B = 0$

$R_A \times 8 - 30 \times 6 - 20 \times 4 - 10 \times 4 \times 2 = 0$

$R_A = 42.5$ kN; so $R_B = 90 - 42.5 = 47.5$kN

SFD:

$SF_A = 42.5$KN

$SF_{AC} = 42.5$KN

$SF_C = 42.5 - 30 = 12.5$KN

$SF_{CD} = 12.5$KN

$SF_D = 12.5 - 20 = -7.5$KN

$SF_E = -47.5$KN

BMD:

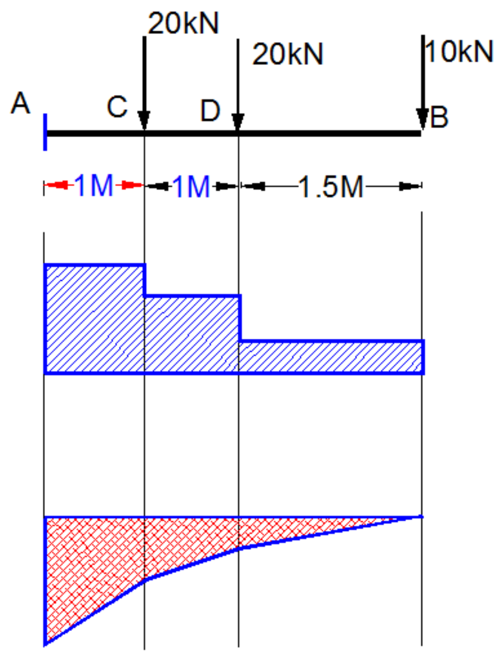
$BM_A = 0$

$BM_C = R_A \times 2 = 85$ KNm

$BM_D = R_A \times 4 - 30 \times 2 = 110$KNm

$BM_B = 0$

Pbm2. Draw the BMD and SFD for the cantilever beam as shown in figure.



$SF_C = 30 + 20 = 50$KN

$SF_A = 50$ KN

BMD:

$BM_B = 0$

$BM_D = 10 \times 1.5 = 15$ KNm

$BM_C = 10 \times 2.5 + 20 \times 1 = 45$ KNm

$BM_A = 10 \times 3.5 + 20 \times 2 + 20 \times 1 = 95$ KNm

SFD:

$SF_B = 10$ KN

$SF_{DB} = 10$ KN

$SF_D = 10 + 20 = 30$KN

$SF_{CD} = 30$ KN

Pbm3. The simply supported beam shown in figure carries two concentrated loads and a uniformly distributed load. Draw the SFD and BMD.